MAT 151 — Chapter 4

Probability Distributions

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Random Variable

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- A *continuous random variable* has as values a non-trivial interval of real numbers (or the union of several such intervals).

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- $\sum_x P(x) = 1$ adding over all possible values of x
- $0 \leq P(x) \leq 1$ for all possible values of x

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$$\sum_x P(x) = P(1) + P(2) + P(3) = \frac{1}{5} + \frac{2}{5} + \frac{3}{5} = \frac{6}{5} \neq 1$$

Measures

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- $\sigma = \sqrt{\left(\sum_x (x^2 \cdot P(x))\right) - \mu^2}$

Example

x	$P(x)$
0	$\frac{1}{512}$
1	$\frac{9}{512}$
2	$\frac{9}{128}$
3	$\frac{21}{128}$
4	$\frac{63}{256}$
5	$\frac{63}{256}$
6	$\frac{21}{128}$
7	$\frac{9}{128}$
8	$\frac{9}{512}$
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  - All outcomes of a trial are classified into *two* categories, success and failure.
  - The probabilities (for success and failure) must remain constant for each trial.

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- The random variable X is the number of successes. It can have the specific value $x = 0, 1, \ldots, n$.
- $P(x)$ is the probability of exactly x successes in the n trials. (And therefore exactly $n - x$ failures.)

Binomial Prob. Dist.

The probability distribution resulting from a binomial experiment is called the *Binomial Probability Distribution*.

Sampling

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In fact the events can also not be considered independent for samples sizes that are smaller if the probability of success or failure is less than 10%.

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- $$P = 0.031$$

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$$P(x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$$

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$$= \binom{7}{4} \cdot 0.2^4 \cdot 0.8^3 + \binom{7}{5} \cdot 0.2^5 \cdot 0.8^2 + \binom{7}{6} \cdot 0.2^6 \cdot 0.8^1 + \binom{7}{7} \cdot 0.2^7 \cdot 0.8^0$$

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$$= 0.033$$

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$$1 - (P(4) + P(5) + P(6) + P(7))$$

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$$P(0) + P(1) + P(2) + P(3)$$

$$1 - (P(4) + P(5) + P(6) + P(7))$$

$$= 1 - 0.033$$

$$= 0.967$$

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- $\sigma = \sqrt{n \cdot p \cdot (1 - p)}$

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The Ericsson method of gender selection *supposedly* has a 75% success rate.
Suppose 100 couples use the Ericson method, with the result that among 100 babies, there are 75 girls.

Example

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  - $\mu = 100 \cdot 0.5 = 50$
  - $\sigma^2 = 100 \cdot 0.5 \cdot (1 - 0.5) = 25$

Example

- Assuming that the Ericsson method has no effect, and assuming that boys and girls are equally likely, find the mean and standard deviation for the number of girls in groups of 100 babies.
    - $\mu = 100 \cdot 0.5 = 50$
    - $\sigma^2 = 100 \cdot 0.5 \cdot (1 - 0.5) = 25$
    - $\sigma = \sqrt{25} = 5$

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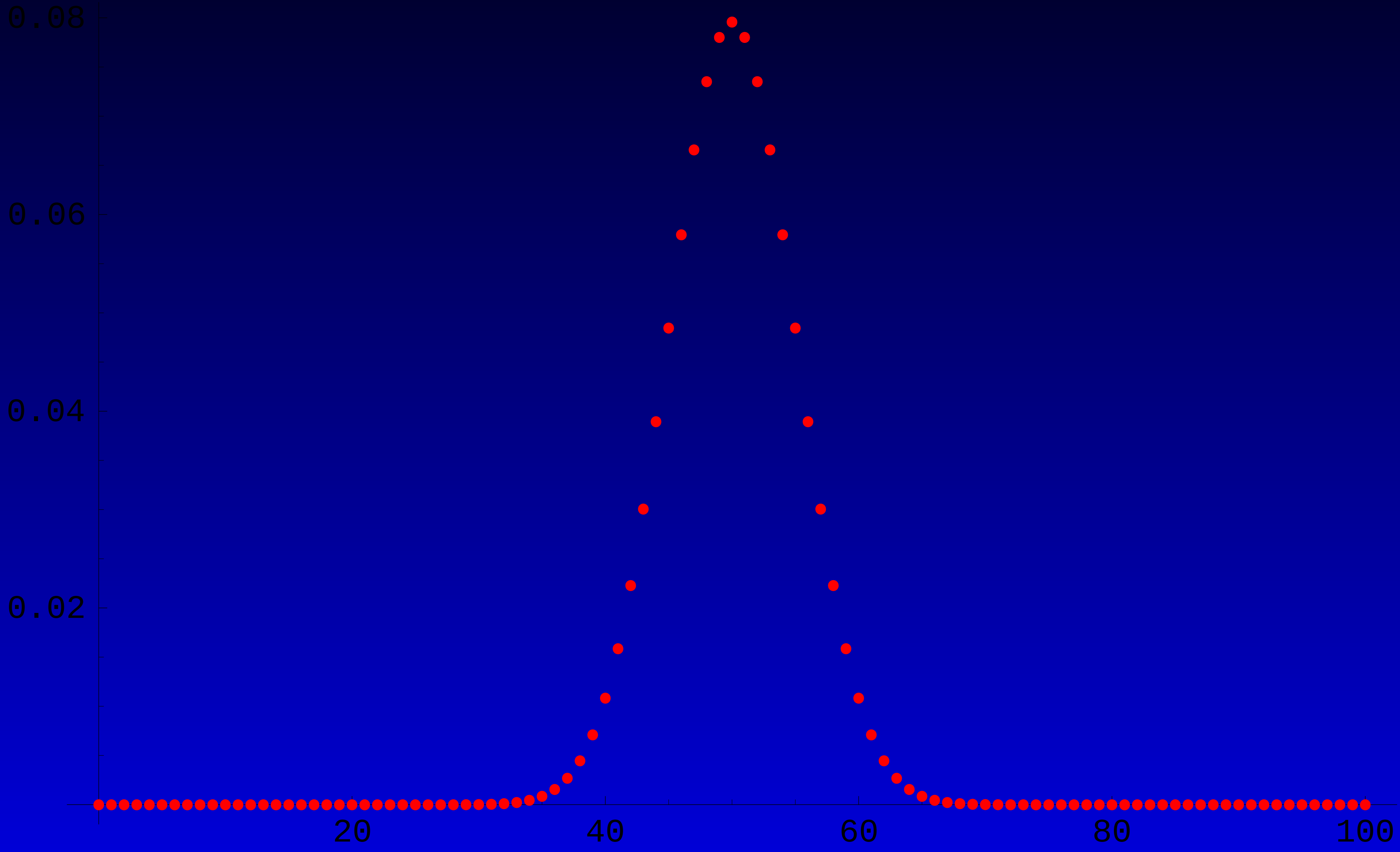
Example

- Interpret the values obtained for μ and σ to determine whether the result of 75 girls among 100 babies supports a claim that the Ericsson method is effective.
  - Any observed values above $\mu + 2\sigma = 60$ or below $\mu - 2\sigma = 40$ are unusual.
  - Since our observation gives a value of 75, it appears that this observation would be extremely unlikely if the Erickson method had no effect.

Example



Example

The probability of obtaining *75 or more* girls in a sample of 100 is

$$2.81814 \cdot 10^{-7} = 0.000000281814$$

Poisson Distribution

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- The *Poisson Distribution* is a discrete probability distribution that applies to occurrences of some event over a specified interval.
- The random variable X is the number of occurrences of the event in an interval.
- The probability of the event occurring x times is

$$P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!}$$

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- The random variable X is the number of occurrences of an event over some interval.
- The occurrences must be random.
- The occurrences must be independent from each other.
- The occurrences must be uniformly distributed over the interval.

Poisson Distribution

- The Poisson distribution has the single parameter μ (which is also the mean of this distribution).

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- The Poisson distribution has the single parameter μ (which is also the mean of this distribution).
- Moreover $\sigma = \sqrt{\mu}$.

Poisson Distribution

The Poisson Distribution was named after *Siméon-Denis Poisson*, a mathematician and physicist born in 1781.

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- Data for 14 corps were combined for the 20 year period, and the 280 corps-years included a total of 196 deaths.
- $\mu = \frac{196}{280}$

Example

- Find the probability that a randomly selected corps-year has the following number of deaths:

# of deaths	0	1	2	3
probability				

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- $P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!}$
- $P(x) = \frac{\left(\frac{196}{280}\right)^x \cdot e^{-\left(\frac{196}{280}\right)}}{x!}$

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- $P(x) = \frac{\left(\frac{196}{280}\right)^x \cdot e^{-\left(\frac{196}{280}\right)}}{x!}$
- $P(0) = \frac{\left(\frac{196}{280}\right)^0 \cdot e^{-\left(\frac{196}{280}\right)}}{0!}$

Example

- Find the probability that a randomly selected corps-year has the following number of deaths:

# of deaths	0	1	2	3
probability	0.497			

- $P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!}$
- $P(x) = \frac{\left(\frac{196}{280}\right)^x \cdot e^{-\left(\frac{196}{280}\right)}}{x!}$
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- Find the probability that a randomly selected corps-year has the following number of deaths:

# of deaths	0	1	2	3
probability	0.497	0.348	0.122	0.0284

- $P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!}$
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Example

- Find the probability that a randomly selected corps-year has the following number of deaths:

# of deaths	0	1	2	3
probability	0.497	0.348	0.122	0.0284
frequency	0.514	0.325	0.114	0.0393

- $P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!}$
- $P(x) = \frac{\left(\frac{196}{280}\right)^x \cdot e^{-\left(\frac{196}{280}\right)}}{x!}$
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