Common Pre-Tasks

1. From Matlab:

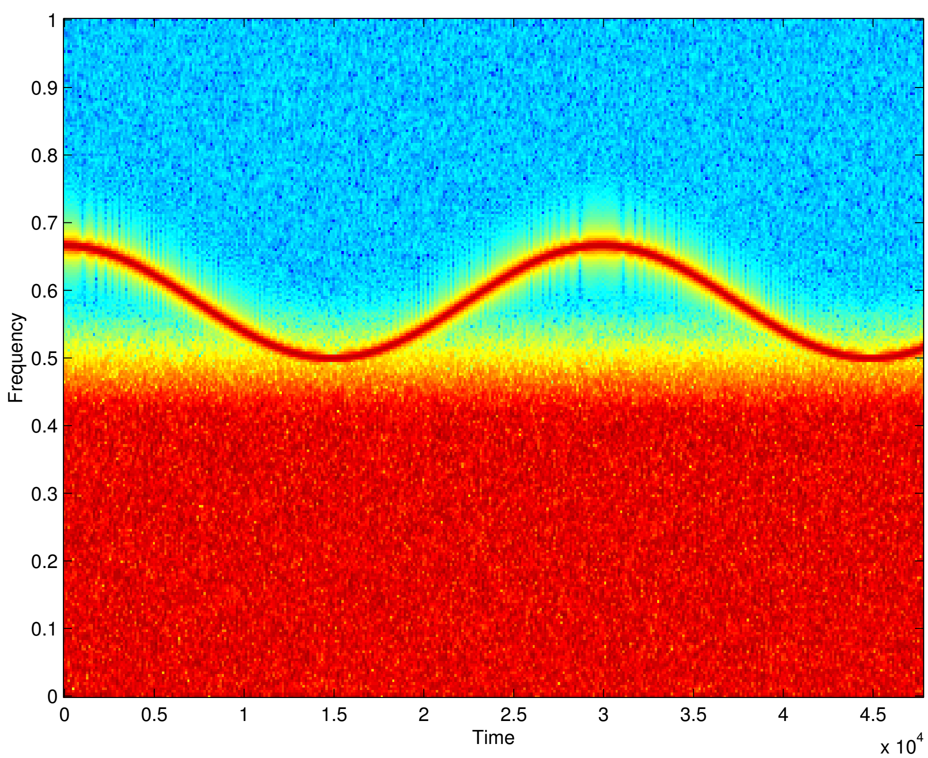
```
>> [y,fs,nbits]=wavread('noisy.wav');
>> length(y)
ans =
       96000
>> fs
fs =
       12000
```

   Thus, there are 96000 samples in the (updated) sound file. The sampling rate is 12000 samples/second. Therefore:

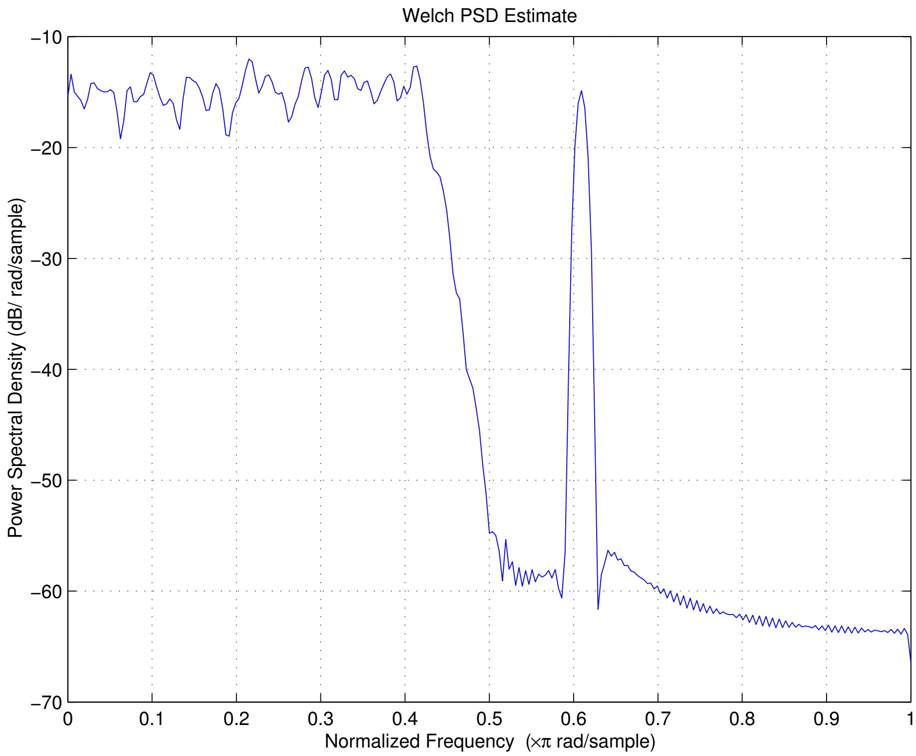
```
>> length(y)/fs
ans =
     8
```

   There are 8 seconds of sound.

2. `>> specgram(y,512)`



```
>> y1=y(47400:48600);
>> pwelch(y1,[],[],512)
```



From the spectrogram, it looks like there is a tone that goes up and down and up and down with a lot of noise underneath that signal. The rising and falling tone goes between the frequencies of around 6kHz to 8kHz. The noise is spread-spectrum between 0Hz and around 5kHz, which can be seen from on the graph of power spectral density.

3. The sound file contains a tone that rises and falls in frequency twice over. It is drowned out by a lot of noise that sounds a lot like a jet engine. This is consistent with what is seen on the spectrogram as well as the noise visible on the power spectral density graph.

4. The highpass filter we will be designing allows high frequencies to pass. This will filter out a large proportion of the noise from the source while not affecting the oscillating signal.

5.

$$\begin{aligned} A_p &= 1\text{dB} \\ A_s &= 20\text{dB} \\ \omega_p &= 18850 \\ \omega_s &= 10053 \end{aligned}$$

$$\begin{aligned} v_s &= \frac{\omega_p}{\omega_s} \\ &= 1.875 \end{aligned}$$

$$\begin{aligned} \varepsilon^2 &= 10^{0.1A_p} - 1 \\ &= 0.2589 \end{aligned}$$

$$
\begin{aligned}
n &= \lceil \frac{\cosh^{-1}\left[\left(10^{0.1A_s}-1\right)/\varepsilon^2\right]^{\frac{1}{2}}}{\cosh^{-1} v_s} \rceil \\
&= \frac{2.5767}{1.2416} \\
&= 2.0754 \longrightarrow 3
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow H_p(s) &= \frac{K_0}{(s+\sinh(\alpha))\left(s^2-2s\sigma_k+\sigma_k^2+\omega_k^2\right)} \\
\text{where} & \\
K_0 &= G\theta_p(0) \\
\sigma_k &= -\sin\theta_k \sinh\alpha \\
\omega_k &= \cos\theta_k \cosh\alpha \\
\theta_p &= \frac{\pi}{6} \\
\alpha &= \frac{1}{3} sinh^{-1}\left(\frac{1}{\varepsilon}\right)
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \alpha &= 0.47599 \\
\sigma &= -0.24709 \\
\omega &= 0.996
\end{aligned}
$$

$$
\begin{aligned}
\theta_p(0) &= \sinh\alpha\left(\sigma^2+\omega^2\right) \\
&= 0.49131 \\
G &= 1 \\
\Rightarrow K_0 &= 0.49131
\end{aligned}
$$

$$
\Rightarrow H_p(s) = \frac{0.49131}{(s+0.4942)\left(s^2+0.4942s+1.0571\right)}
$$

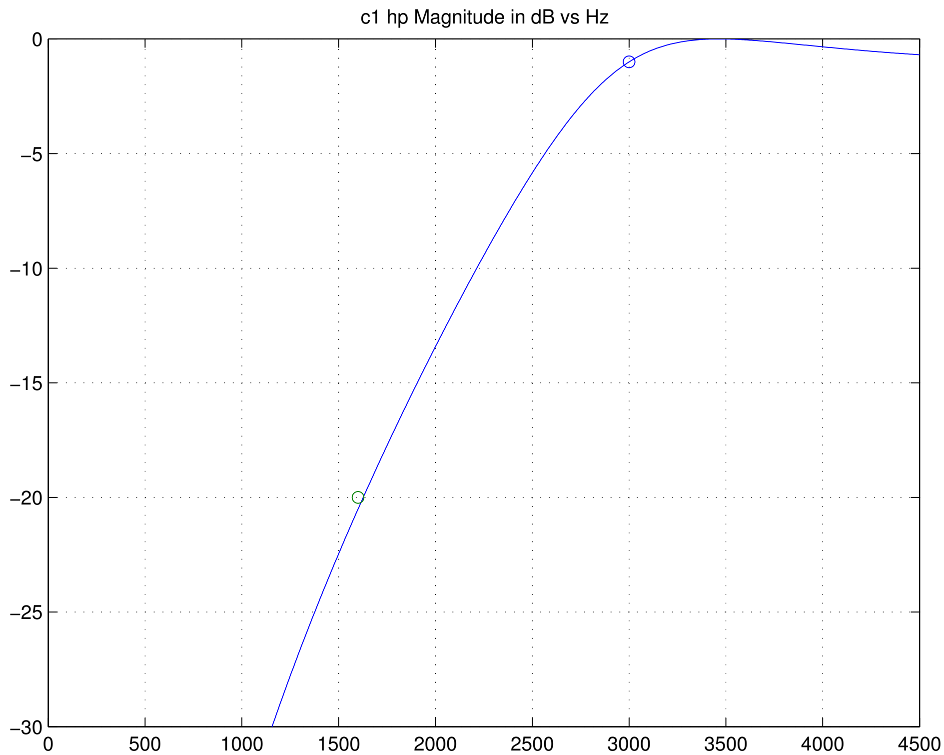
Task 1

1.
```
>> [n,d,np,dp]=afd('c1','hp',[1 20],3000,1600);
>> n
n =
            1            0            0            0
>> d
d =
            1        47513   7.1475e+08   1.3632e+13
>> np
np =
            0            0            0      0.49131
>> dp
dp =
            1      0.98834       1.2384      0.49131
```

Thus, the transfer functions for the prototype and the filter respectively are:

$$
\begin{aligned}
H_p(s) &= \frac{0.49131}{s^3+0.98834s^2+1.2384s+0.49131} \\
H(s) &= \frac{s^3}{s^3+47513s^2+7.1475\times10^8 s+1.3632\times10^{13}}
\end{aligned}
$$

The magnitude response of the filter will be:



The calculated coefficients are practically identical to those calculated in task 2(a), which is what was expected.

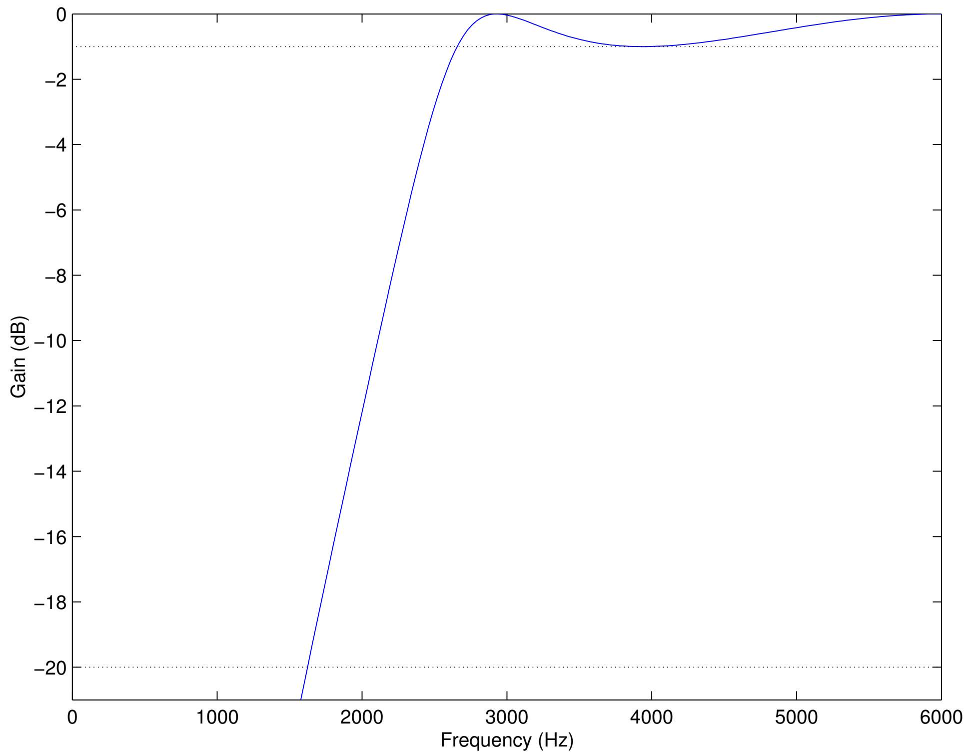
2.
```
>> [nz,dz]=s2zni(n,d,12000,'bili',1600);
>> nz
nz =
          1            -3             3            -1
>> dz
dz =
     5.6903      -0.14992        3.0462       0.88646
```

Thus the transfer function is:

$$H(z) = \frac{1 - 3z^{-1} + 3z^{-2} - z^{-3}}{5.6903 - 0.14992z^{-1} + 3.0462z^{-2} + 0.88646z^{-3}}$$

At first inspection it looks like the coefficients calculated with Matlab are different to the ones calculated in task 3(a). However, inspection shows that dividing each of the coefficients by 5.6903 will give the same transfer function as derived in task 3(a).

3.
```
>> [h p f]=tfplot('z',nz,dz);
>> H=20*log10(h);
>> F=f*12000;
>> plot(F,H, F,-1,'k:', F,-20,'k:');
```



4. As you can see from the plot above, the given filter has no more than 1dB attenuation in the passband (above 3kHz) and that the filter also passes through the 20dB attenuation mark at the beginning of the stopband (1.6kHz). This means that the digital filter is within the specification given.

Task 2

1.

$$
\begin{aligned}
H_p(s) &= \frac{0.49131}{s^3 + 0.98834s^2 + 1.2384s + 0.49131} \\
\text{LP2HP :} & \\
s &\longrightarrow \frac{\omega_p}{s} \\
H(s) &= \frac{0.49131}{6.6974 \times 10^{12} s^{-3} + 3.5116 \times 10^{8} s^{-2} + 23343 s^{-1} + 0.49131} \cdot \frac{s^3}{s^3} \\
&= \frac{0.49131 s^3}{0.49131 s^3 + 23343 s^2 + 35116 \times 10^{8} s + 6.6974 \times 10^{1}2} \div \frac{0.49131}{0.49131} \\
&= \frac{s^3}{s^3 + 47513 s^2 + 7.1475 \times 10^{8} s + 1.3632 \times 10^{13}}
\end{aligned}
$$

2. Using Matlab to factorise the transfer function:

```
d =
            1        47513   7.1475e+08   1.3632e+13
>> r=roots(d)
r =
      -38144
     -4684.6 +      18315i
     -4684.6 -      18315i
>> poly(r(2:3))
ans =
            1       9369.2   3.5738e+08
```

Thus giving:

$$H(s) = \frac{s}{(s+38144)} \cdot \frac{s^2}{(s^2+9369.2s+3.5738\times 10^8)}$$

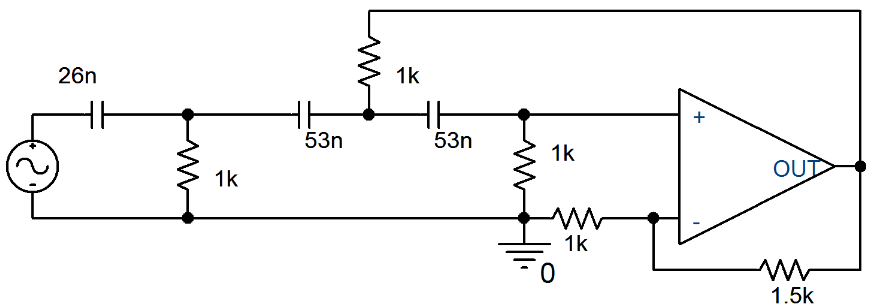
Therefore, for the first order filter:

$$\begin{aligned} \omega_0 &= 38144 \\ R &= 1\text{k}\Omega \\ C &= \frac{1}{R\omega_0} \\ &= 26.2\text{nF} \end{aligned}$$

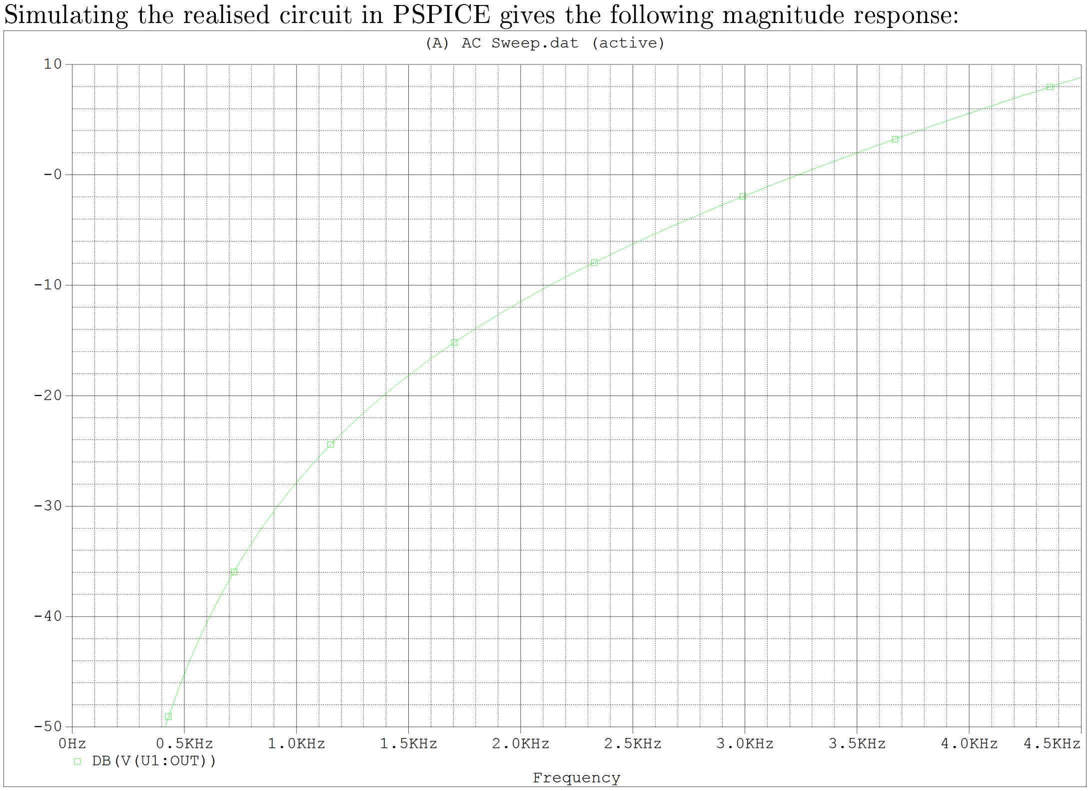
For the second order filter:

$$\begin{aligned} {\omega_0}^2 &= 3.5738\times 10^8 \\ \Rightarrow \omega_0 &= 18904 \\ C &= \frac{1}{R\omega_0} \\ &= 52.9\text{nF} \\ \\ Q &= \frac{\omega_0}{9369.2} \\ &= 2.0177 \\ \\ K &= 3-\frac{1}{Q} \\ &= 2.5044 \\ &= 1+\frac{R_a}{R_b} \\ \Rightarrow R_a &= 1.5\text{k}\Omega \\ R_b &= 1\text{k}\Omega \end{aligned}$$

This is realised as a two stage filter with a single first-order highpass filter and a single second-order Sallen-Key highpass filter. The opamp in the filter is configured to be non-inverting and provide a K of 2.5.



3. Simulating the realised circuit in PSPICE gives the following magnitude response:



Task 3

1.

$$
\begin{aligned}
f_0 &= 1600\text{Hz} \\
S &= 12000 \\
\Omega &= 2\pi f_0/S \\
&= 0.83776 \\
\omega &= C \tan\frac{\Omega}{2} \\
\Rightarrow C &= 22580
\end{aligned}
$$

$$
\begin{aligned}
H(s) &= \frac{s^3}{s^3 + 47513s^2 + 7.1475 \times 10^8 s + 1.3632 \times 10^{13}} \\
s &\longrightarrow C\frac{z-1}{z+1} \\
\Rightarrow H(z) &= \frac{\left(C\frac{z-1}{z+1}\right)^3}{\left(C\frac{z-1}{z+1}\right)^3 + 47513\left(C\frac{z-1}{z+1}\right)^2 + 7.1475 \times 10^8 \left(C\frac{z-1}{z+1}\right) + 1.3632 \times 10^{13}} \\
&= \frac{C^3 (z-1)^3}{C^3 (z-1)^3 + 47513C^2 (z-1)^2 (z+1) + 7.1475 \times 10^8 C (z-1)(z+1)^2 + 1.3632 \times 10^{13} (z+1)^3}
\end{aligned}
$$

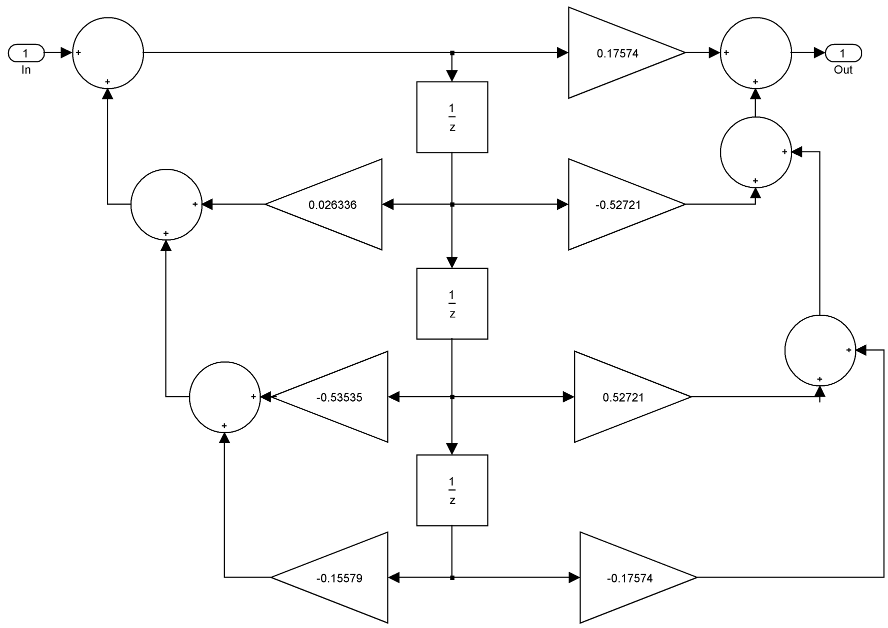
Using Matlab:

```
>> Hd1 = poly([1 1 1])
Hd1 =
     1    -3     3    -1
>> Hd20 = poly([1 1 -1])
Hd20 =
     1    -1    -1     1
>> Hd2 = poly([1 1 -1])
Hd2 =
     1    -1    -1     1
>> Hd3 = poly([1 -1 -1])
Hd3 =
     1     1    -1    -1
>> Hd4 = poly([-1 -1 -1])
Hd4 =
     1     3     3     1
>> Hd1 = Hd1 .* C^3
Hd1 =
   1.1512e+13  -3.4536e+13   3.4536e+13  -1.1512e+13
>> Hd2 = Hd2 .* 47513 * C^2
Hd2 =
   2.4224e+13  -2.4224e+13  -2.4224e+13   2.4224e+13
>> Hd3 = Hd3 .* 7.1475e8 * C
Hd3 =
   1.6139e+13   1.6139e+13  -1.6139e+13  -1.6139e+13
>> Hd4 = Hd4 .* 1.3632e13
Hd4 =
   1.3632e+13   4.0896e+13   4.0896e+13   1.3632e+13
>> Hd = Hd1 + Hd2 + Hd3 + Hd4
Hd =
   6.5507e+13  -1.7252e+12   3.5069e+13   1.0205e+13
>> Hn = poly([1 1 1]) .* C^3
Hn =
   1.1512e+13  -3.4536e+13   3.4536e+13  -1.1512e+13
>> Hn = Hn ./ Hd(1)
Hn =
      0.17574     -0.52721      0.52721     -0.17574
>> Hd = Hd ./ Hd(1)
Hd =
            1    -0.026336      0.53535      0.15579
```

Ergo, the transfer function is:

$$H(z) \quad = \quad \frac{0.17574 - 0.52721z^{-1} + 0.52721z^{-2} - 0.17574z^{-3}}{1 - 0.026336z^{-1} + 0.53535z^{-2} + 0.15579z^{-3}}$$

2. The IIR Direct form II realisation of the filter is:

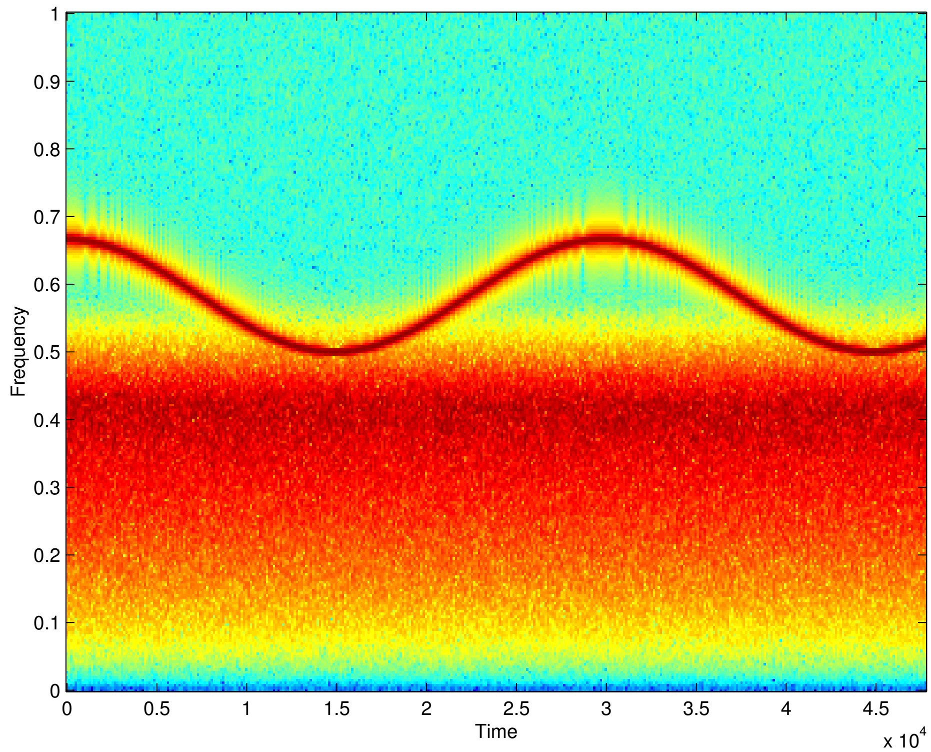


Common Post-Tasks

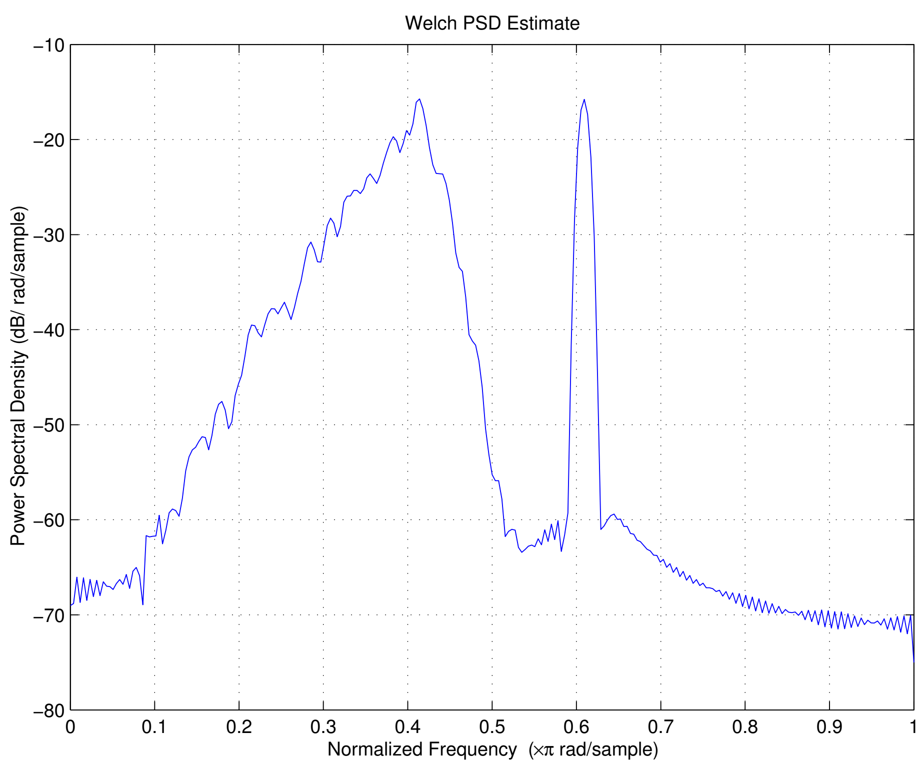
1. Using Matlab with the values of y, Hn and Hd acquired previously:

```
>> o=filter(Hn,Hd,y);
```

2. >> specgram(o,512)




```
>> o1=o(47400:48600);
>> pwelch(o1,[],[],512)
```



From both the spectrogram and the power spectral density graph we can see that there is a decrease in the intensity of the noise in the filtered signal. This is most obvious for frequencies below 1.6kHz (0.133 cycles/sample), however there is also a noticeable decrease in noise in the region between the passband and the stop band (up to 0.25 cycles/sample). The colour of the spectrogram in this region has gone from dark red in the original source to orange/yellow in the filtered version. This indicates a drop in the intensity of the noise.

From the power spectral density graph we can see that around half of the power of the noise has been filtered out.

3. The noise is less obvious in the filtered output. The bassy, "jet engine-like" noise is no longer present, however the noise is still noticeable as very quiet white noise. The signal (the oscillating pitch) remains unaffected.

   This decrease in the intensity of the noise can be seen from the decrease in the power spectral density for the noise (from the graph) and the decrease in colour intensity in the noiseband on the spectrogram.

4. The original sound was very noisy. A lot of the noise has been removed in the filtered sample, but not all of it. The filter has been of benefit in removing that large portion of the noise from the original sample. The noise in the sample is no longer as overwhelming and the oscillating tone can now be better heard.

   A filter which had a higher frequency passband edge could conceivably have also distorted the signal we wished to preserve. A filter with a sharper transition between the stopband and passband would have to be of higher order and therefore be more expensive to realise physically or digitally.