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## C O



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## 2 B

### 2 .

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```
{ a, b, c, d }::Indices.
A_{a b} B_{b c};
```

$A_a$ $B_b$ ; $b$ $c$ (

```
@substitute!(%)( B_{a b} ->  C_{a b c} D_{c} );
```

$A_a$ $C_b$ $D_d$; $b$ $c$ $d$ (

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```
A_{\dot{a} \dot{b}}::AntiSymmetric.
A_{\dot{b} \dot{a}};
```

$A_{\dot{b}}$ ; . (

```
@canonicalise!(%);
```

( $\cdot_{\dot{b}}$; (

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```
{ a_{1}, a_{2}, a_{3}, a_{4} }::Indices(vector).
V_{a_{1}} W_{a_{1}}:
@substitute!(%)( V_{a_{2}} -> M_{a_{2} a_{1}} N_{a_{1}} );
```

$M_{a_1 a_2} N_{a_2} W_{a_1};$ (

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```
R_{a b c d}::TableauSymmetry(shape={2,2}, indices={0,2,1,3}).
R_{a b c d} R_{d c a b}:
@canonicalise!(%);
```

( $a$ $R_a$ ; $b$ $b$ ( $c$

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```
\nabla{#}::Derivative.
\partial{#}::PartialDerivative.
A_{m n}::AntiSymmetric.
V_{m}::Depends(\nabla).

\partial_{m p}( A_{q r} V_{n} ) A^{p m};
```

$\partial_{m} ( \, _{q} V_{n}) \; ^{p}$ ; $r$ ( $p$ $m$

```
@canonicalise!(%);
```

0; (

```
\nabla_{m p}( A_{q r} V_{n} ) A^{p m};
@canonicalise!(%);
```

( $_{m}$ ( $_{q}$ $V_{n}$) $^{m}$ ; $r$ ( $p$

```
@unwrap!(%);
```

( $_{q}$ $\nabla_{m}$ $V_{n} A^{m}$ ; $r$ ( $p$

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```
{m,n,p,q,r,s,t#}::Indices(vector).
\nabla{#}::Derivative.
R_{m n p q}::RiemannTensor.
\nabla_{m}{R_{p q r s}}::SatisfiesBianchi.
```

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```
\nabla_{m}{R_{p q r s}} + \nabla_{p}{R_{q m r s}} + \nabla_{q}{R_{m p r s}}:
@young_project_tensor!2(%){ModuloMonoterm}:
@collect_terms!(%);
```

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$$A_m \quad B_m \quad - \quad {}_m \quad B_m \quad = \qquad ( \quad {}_n \quad {}_n \quad {}_n$$

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```
{ m, n, p, q }::Indices(vector).
{ A_{m n p}, B_{m n p} }::AntiSymmetric.
A_{m n p} B_{m n q} - A_{m n q} B_{m n p};
```

$$A_m \quad B_m \quad - \quad {}_m \quad B_m \quad ; \qquad ( \quad {}_n \quad {}_n \quad {}_n$$

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```
{ m, n, p, q }::Integer(1..4).
@decompose_product!(%):
@canonicalise!(%):
@collect_terms!(%);
```

$$A_p \quad B_q \quad - \quad {}_q \quad B_p \quad ; \quad {}_m \quad {}_m \qquad ( \quad {}_m$$

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```
{ m, n, p, q }::Integer(1..3).
@decompose_product!(%):
@canonicalise!(%):
@collect_terms!(%);
```

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```
\tableau{#}::FilledTableau(dimension=10).
\tableau{0,0}{1,1} \tableau{a,a}{b,b}:
@lr_tensor!(%);
```

$$\begin{array}{|c|c|c|c|}\hline 0&0&a&a\\\hline 1&1&b&b\\\hline\end{array} + \begin{array}{|c|c|c|c|}\hline 0&0&a&a\\\hline 1&1&b\\\cline{1-3} b\\\cline{1-1}\end{array} + \begin{array}{|c|c|c|c|}\hline 0&0&a&a\\\hline 1&1\\\cline{1-2} b&b\\\cline{1-2}\end{array} + \begin{array}{|c|c|c|}\hline 0&0&a\\\hline 1&1&b\\\hline a\\\cline{1-1} b\\\cline{1-1}\end{array} + \begin{array}{|c|c|c|}\hline 0&0&a\\\hline 1&1\\\cline{1-2} a&b\\\cline{1-2} b\\\cline{1-1}\end{array} + \begin{array}{|c|c|}\hline 0&0\\\hline 1&1\\\hline a&a\\\hline b&b\\\hline\end{array};$$

```
@tabdimension!(%);
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$$E_i = {}_i{}^m\ C_j\ \ C_l{}^p + \frac{1}{4}C_i{}^m\ C_j\ \ C_k{}^p - \frac{1}{2}C_i\ \ C^k\ \ C^l{}_m\ \ ,_l \quad {}_j{}^m \quad {}_p \quad {}^k$$ (

$$E \quad {}_j \quad C^m \quad C_p{}^j{}_q + \frac{1}{2}C_j = C^p \quad C^j{}_p\ . \quad {}_m \quad {}^k \quad {}^p$$

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$$\nabla_i\nabla_j E_i - \frac{1}{6}\nabla_i\nabla_i E \qquad = $$ ( $j$

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```
{i,j,m,n,k,p,q,l,r,r#}::Indices(vector).
C_{m n p q}::WeylTensor.
\nabla{#}::Derivative.
\nabla_{r}{ C_{m n p q} }::SatisfiesBianchi.

Eij:=- C_{i m k l} C_{j p k q} C_{l p m q} + 1/4 C_{i m k l} C_{j m p q} C_{k l p q}
      - 1/2 C_{i k j l} C_{k m p q} C_{l m p q}:

E:=  C_{j m n k} C_{m p q n} C_{p j k q} + 1/2 C_{j k m n} C_{p q m n} C_{j k p q}:

exp:= \nabla_{i}{\nabla_{j}{ @(Eij) }} - 1/6 \nabla_{i}{\nabla_{i}{ @(E) }}:
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@distribute!(%): @prodrule!(%):
@distribute!(%): @prodrule!(%):

@prodsort!(%): @canonicalise!(%): @rename_dummies!(%):
@collect_terms!(%):
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@substitute!(%)( \nabla_{i}{C_{k i l m}} -> 0 ), \nabla_{i}{C_{k m l i}} -> 0 );
```

$$
\begin{aligned}
& C_i \quad C_i \quad \nabla_q \nabla_j C_n \quad - \quad {}_i \quad \nabla_k C_i \quad \nabla_p C_j \quad - \quad {}_i \quad \nabla_i C_m \quad \nabla_p C_j \qquad {}_j \quad {}_k \quad {}_k \\
& - \quad {}_i \quad C_i \quad \nabla_m \nabla_p C_j \quad - \frac{1}{4} C_i \quad C_i \quad \nabla_q \nabla_m C_n \quad + \frac{1}{4} C_i \quad \nabla_k C_i \quad \nabla_p C_m \qquad {}_j \quad {}_k \quad {}_q \\
& - \frac{1}{2} C_i \quad \nabla_i C_j \quad \nabla_k C_m \quad + \frac{1}{4} C_i \quad C_i \quad \nabla_j \nabla_k C_m \quad + \frac{1}{2} C_i \quad C_i \quad \nabla_m \nabla_j C_n \qquad {}_j \quad {}_k \\
& + \frac{1}{2} C_i \quad \nabla_i C_m \quad \nabla_n C_j \quad - \frac{1}{2} C_i \quad \nabla_i C_j \quad \nabla_m C_n \quad + \frac{1}{2} C_i \quad C_i \quad \nabla_j \nabla_m C_n \qquad ( {}_j \quad {}_k \quad {}_k \\
& + \frac{1}{2} C_i \quad C_i \quad \nabla_q \nabla_q C_j \quad + \quad {}_i \quad \nabla_k C_i \quad \nabla_k C_j \quad - \frac{1}{4} C_i \quad C_i \quad \nabla_q \nabla_q C_m \qquad {}_j \quad {}_k \quad {}_k \\
& - \frac{1}{2} C_i \quad \nabla_k C_i \quad \nabla_k C_m \quad ; \qquad {}_j \quad {}_j
\end{aligned}
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@young_project_product!(%):
@sumflatten!(%):
@collect_terms!(%);
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```
{M, N, P}::Indices(space).
{m, n, p}::Indices(subspace1).
{a, b, c}::Indices(subspace2).

A_{M N} B_{N P};
@split_index!(%){M, m, a};
```

$A_M \quad B_m \; + \quad _M \; B_a \quad ; \qquad _m \qquad Pa \qquad ( \qquad P$

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$$g_{\mu} \; = \begin{pmatrix} \phi^{-} \quad h_m \; + \quad _\nu \; {}_m A_n & \phi \quad _m \\ \phi \quad _n & \phi \end{pmatrix} , \qquad ( \qquad n$$

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$$R_m \;=\; \frac{1}{2}\nabla_m\partial_n\phi \quad \frac{1}{4}\partial_m\phi \;\; _n\phi \;\; ^{-} + \frac{1}{4}\partial_p\phi \;\; _q\phi \;\; ^{-} \; h_m \;\; h^p \; + \frac{1}{4}F_m \;\; F_n \;\; \phi^3 h^p \;\; , \qquad ( \quad 4$$

w $_m$ i h $_m$. $n$

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```
{\mu,\nu,\rho,\sigma,\kappa,\lambda,\eta,\chi#}::Indices(full, position=fixed).
{m,n,p,q,r,s,t,u,v,m#}::Indices(subspace, position=fixed, parent=full).
```

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```
\partial{#}::PartialDerivative.
g_{\mu\nu}::Metric.
g^{\mu\nu}::InverseMetric.
g_{\mu? \nu?}::Symmetric.
g^{\mu? \nu?}::Symmetric.
```

```
h_{m n}::Metric.
h^{m n}::InverseMetric.
\delta^{\mu?}_{\nu?}::KroneckerDelta.
\delta_{\mu?}^{\nu?}::KroneckerDelta.
F_{m n}::AntiSymmetric.
```

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```
RtoG:= R^{\lambda?}_{\mu?\nu?\kappa?} ->
 - \partial_{\kappa?}{ \Gamma^{\lambda?}_{\mu?\nu?} }
 + \partial_{\nu?}{ \Gamma^{\lambda?}_{\mu?\kappa?} }
 - \Gamma^{\eta}_{\mu?\nu?} \Gamma^{\lambda?}_{\kappa?\eta}
 + \Gamma^{\eta}_{\mu?\kappa?} \Gamma^{\lambda?}_{\nu?\eta}:

Gtog:= \Gamma^{\lambda?}_{\mu?\nu?} ->
   (1/2) * g^{\lambda?\kappa} (
      \partial_{\nu?}{ g_{\kappa\mu?} } + \partial_{\mu?}{g_{\kappa\nu?} }
                                    - \partial_{\kappa}{ g_{\mu?\nu?} } ):
```

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todo:= g_{m1 m} R^{m1}_{4 n 4} + g_{4 m} R^{4}_{4 n 4};
@substitute!(%)( @(RtoG) );
@substitute!(%)( @(Gtog) );
@distribute!(%);
@prodrule!(%);
@distribute!(%);
@prodsort!(%);
```

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@split_index!!(%){\mu,m1,4};
@canonicalise!(%):
@substitute!(%)( \partial_{4}{A??} -> 0 );
@substitute!(%)( \partial_{4 m?}{A??} -> 0 );
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```
@substitute!(%)( g_{4 4} -> \phi ):
@substitute!(%)( g_{4 m} -> \phi A_{m} ):
@substitute!(%)( g_{m n} -> \phi**{-1} h_{m n} + \phi A_{m} A_{n} );
@substitute!(%)( g^{4 4} -> \phi**{-1} + \phi A_{m} h^{m n} A_{n} ):
@substitute!(%)( g^{4 m} -> - \phi h^{m n} A_{n} ):
```

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```
@substitute!(%)( g^{m n} -> \phi h^{m n} ):
@distribute!(%):
@prodrule!(%):
@distribute!(%):
@prodrule!(%);
@distribute!(%);
@canonicalise!(%):
@substitute!!(%)( h_{m1 m2} h^{m3 m2} -> \delta_{m1}^{m3} );
@eliminate_kr!(%):
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```
@substitute!(%)( \partial_{m}{\phi**{-1}} -> -\phi**{-2} \partial_{m}{\phi} ):
@collect_factors!(%):
@prodsort!(%):
@substitute!(%)( \partial_{p}{h^{n m}} h_{q m} -> - \partial_{p}{h_{q m}} h^{n m} );
@canonicalise!(%):
@substitute!(%)( h_{m1 m2} h^{m3 m2} -> \delta_{m1}^{m3} );
@eliminate_kr!(%);
```

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```
@substitute!(%)( \partial_{n}{A_{m}} -> 1/2*\partial_{n}{A_{m}} + 1/2*F_{n m}
                                       + 1/2*\partial_{m}{A_{n}} ):
@distribute!(%):
@prodsort!(%):
@canonicalise!(%):
@rename_dummies!(%):
@collect_terms!(%);
```

$$-\frac{1}{4}\partial_m\phi \;{}_n\phi \;{}^- \;+\frac{1}{4}\partial_p\phi \;{}_n h_m \; h^p \;-\frac{1}{2}\partial_m \;\phi \quad \frac{1}{4}F_m \; F_n \; \phi^3 h^p$$
$$+\frac{1}{4}\partial_p\phi \;{}_q\phi \;{}^- \; h_m \; h^p \;-\frac{1}{4}\partial_p\phi \;{}_q h_m \; h^p \;+\frac{1}{4}\partial_p\phi \;{}_m h_n \; h^p \;; \qquad (\,$$

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::PostDefaultRules( @@prodsort!(%), @@eliminate_kr!(%),
                                    @@canonicalise!(%), @@collect_terms!(%) ).
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```
{s,r,l,k,m,n}::Indices(vector).
{s,r,l,k,m,n}::Integer(0..d-1).
\Gamma_{#}::GammaMatrix(metric=\delta).
\delta_{m n}::KroneckerDelta.
```

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```
\Gamma_{s r} \Gamma_{r l} \Gamma_{k m} \Gamma_{m s}:
@join!(%){expand}:
@join!(%){expand};
```

$( \quad _l - {}_l\, d \quad _l\, d \quad _l\, r) \quad _k\, r - {}_k\, d \quad _k - {}_k\, d \quad _r \quad _r \quad _r \quad _r \quad ( \; + \; -$

```
@distribute!(%);
@join!(%){expand};
@distribute!(%);
@factorise!(%){d};
@collect_factors!(%);
```

$\Gamma_k\, ( \quad {}^2 - {}^3) \quad _{lk}\, ( \quad {}^2 + {}^3) \quad _l \quad ($

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{a,b,c,d#}::Indices(spinor).
\Gamma_{#}::GammaMatrix(metric=\delta).
(\Gamma_{m n})_{a b} (\Gamma_{n p})_{b c};
@combine!(%);
```

$$( \;_{m}\; \Gamma_{n}\; )_{a}\; ; \quad c \quad p \quad ( \quad n$$

```
@join!(%){expand}:
@canonicalise!(%);
```

$$( \;_{m}\; \delta_{n} - \;_{m}\; \delta_{n} + \;_{p}\; \delta_{m} + \;_{m}\; \delta_{n} - \;_{m}\; \delta_{n}\; )_{a}\; ; \quad n \quad (_{c} \;_{p}\;_{p} \quad n$$

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@distribute!(%):
@expand!(%);
```

$$( \;_{m}\; )_{a}\; \delta_{n} - \;_{m}\; )_{a}\; \delta_{n} + \;_{p}\; )_{a}\; \delta_{m} + \;_{m\,c}\delta_{n}\; )_{a} - \;_{m}\; \delta_{n}\; )_{a}\; ; \quad c \quad n \quad c \quad p \quad p \quad c$$

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$$- \;_{[}\; {}^{s}(\bar\theta \;{}^{r}\; \psi_{\rho})\; \bar\psi_{\mu}\,\Gamma^{r}_{\nu}\epsilon \quad \sum_{n} \frac{1}{2^5\; n} (\bar\psi_{\mu}\Gamma^{i_1\cdot}\;{}^{n}\psi_{\rho})\; \bar\theta \;{}^{r}\; \Gamma^{i_n\cdot\cdot}\;{}^{1}\Gamma^{r}\epsilon \quad !$$

$$= \frac{1}{2^5}\bar\psi_{[}\; \Gamma_{m}\psi_{\rho}e_{\nu}\;{}^{s}\left(8\bar\theta\;{}^{s}_{\mu}\;\epsilon \quad {}^{s}\;\bar\theta\;\right) \quad m \quad m \quad ($$

$$- \frac{1}{2^5\; 2!}\bar\psi_{[}\; \Gamma_{m}\; \psi_{\rho}e_{\nu}\;{}^{s}\left(-_{\mu}\bar\theta\;{}^{s}\;\epsilon \quad \bar\theta\;{}^{[}\;\epsilon\;{}^{n}\right) \quad m \quad m$$

$$+ \frac{1}{2^5\; 5!}\bar\psi_{[}\; \Gamma_{m}\; \psi_{\rho}e_{\nu}\;{}^{s}\left(_{\mu}10\eta^{s}\;\bar\theta\;{}^{n}\;\epsilon\right). \quad [ \quad m$$

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```
{\mu,\nu,\rho}::Indices(curved, position=fixed).
{m,n,p,q,r,s,t#}::Indices(flat).
{m,n,p,q,r,s,t#}::Integer(0..10).
T^{#{\mu}}::AntiSymmetric.
\psi_{\mu}::SelfAntiCommuting.
\psi_{\mu}::Spinor(dimension=11, type=Majorana).
\theta::Spinor(dimension=11, type=Majorana).
\epsilon::Spinor(dimension=11, type=Majorana).
{\theta,\epsilon,\psi_{\mu}}::AntiCommuting
\bar{#}::DiracBar.
\delta^{m n}::KroneckerDelta.
\Gamma^{#{m}}::GammaMatrix(metric=\delta).
```

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```
T^{\mu\nu\rho} e_{\nu}^{s}
    \bar{\theta} \Gamma^{r s} \psi_{\rho}
    \bar{\psi_{\mu}} \Gamma^{r} \epsilon;

@fierz!(%)( \theta, \epsilon, \psi_{\mu}, \psi_{\nu} );
```

$$-\frac{1}{32}T^{\mu} \quad e_{\nu}^{s}\bar{\theta} \quad {}^{r}\ \Gamma^{r}\epsilon\bar{\psi}_{\mu}\psi_{\rho} - {}^{\nu}\frac{1}{32}T^{\mu} \quad e_{\nu}^{s}\bar{\theta} \quad {}^{r}\ \Gamma^{m}\Gamma^{r}\epsilon\bar{\psi}_{\mu}\Gamma^{\nu}_{m}\psi_{\rho} \quad {}^{s} \quad {}^{s}$$
$$-\frac{1}{64}T^{\mu} \quad e_{\nu}^{s}\bar{\theta} \quad {}^{r}\ \Gamma^{m} \quad \Gamma^{r}\epsilon\bar{\psi}_{\mu}\Gamma_{n} \quad \psi_{\rho} - \frac{1}{192}T^{\mu} \quad e_{\nu}^{s}\bar{\theta} \quad {}^{r}\ \Gamma^{m} \quad \Gamma^{r}\epsilon\bar{\psi}^{\nu}_{\mu}\Gamma_{p}{}^{s} \quad \psi_{\rho} \quad ( \quad {}^{s} \quad {}_{m}$$
$$-\frac{1}{768}T^{\mu} \quad e_{\nu}^{s}\bar{\theta} \quad {}^{r}\ \Gamma^{m} \quad \Gamma^{r}\epsilon\bar{\psi}_{\mu}\Gamma_{q} \quad \psi_{\rho} - \frac{1}{3840}T^{\mu} \quad e_{\nu}^{s}\bar{\theta} \quad {}^{r}\ \Gamma^{m} \quad \Gamma^{r}\epsilon\bar{\psi}_{\mu}\Gamma_{t} \quad \psi_{\rho};$$

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```
@join!(%){expand}:
@distribute!(%):
@eliminate_kr!(%):
@join!(%){expand}:
@distribute!(%):
@eliminate_kr!(%):
@collect_terms!(%):
@canonicalise!(%):
@rename_dummies!(%):
@collect_terms!(%);
```

$$\frac{1}{4}T^{\mu} \quad e_{\mu}{}^{m}\bar{\theta} \quad {}^{m} \quad \epsilon\bar{\psi}_{\nu}\Gamma_{n}\psi_{\rho} + \frac{5}{16}T^{\mu} \quad e_{\mu}{}^{m}\bar{\theta} \quad \bar{\psi}_{\nu}\Gamma_{m}\psi_{\rho} + {}^{\nu}\frac{3}{32}T^{\mu} \quad e_{\mu}{}^{m}\bar{\theta} \quad {}^{m} \quad \epsilon\bar{\psi}_{\nu}\Gamma_{n}{}^{\nu}\psi_{\rho} \quad ( \quad {}^{n}$$
$$+\frac{1}{4}T^{\mu} \quad e_{\mu}{}^{m}\bar{\theta} \quad {}^{n}\epsilon\bar{\psi}_{\nu}\Gamma_{m} \quad \psi_{\rho} + \frac{1}{384}T^{\mu} \quad e_{\mu}{}^{m}\bar{\theta} \quad {}^{n} \quad \epsilon\bar{\psi}_{\nu}\Gamma_{m}{}^{\nu} \quad \psi_{\rho};$$

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```
\psi_{\mu}::SelfAntiCommuting.
{ \chi, \psi_{\mu} }::AntiCommuting.
\chi A^{\mu\nu} \psi_{\mu} \chi \psi_{\nu};
```

$\chi$ $^{\mu}$ $\psi_{\mu}\chi$ $_{\nu}$; $^{\nu}$ (

```
@substitute!(%)( \psi_{\mu} \psi_{nu} -> B_{\mu\nu} );
```

( $^{\mu}$ $B_{\mu}$ $\chi$ $^{\nu}$ $_{\nu}$ (

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@pop(%);
```

$\chi$ $^{\mu}$ $\psi_{\mu}\chi$ $_{\nu}$; $^{\nu}$ (

Now $^{\mu}$ t w$^{\nu}$ e

```
A^{\mu\nu}::Symmetric.
@canonicalise!(%);
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```
::PostDefaultRules( @@eliminate_kr!(%), @@prodsort!(%), @@collect_terms!(%) ).
```

```
D{#}::Derivative.
\bar{#}::DiracBar.
\delta{A??}::Derivative.
{m,n,p,q,r,s,t,u,m#}::Indices(flat).
{m,n,p,q,r,s,t,u,m#}::Integer(0..3).
{\mu,\nu,\rho,\sigma,\kappa,\lambda,\alpha,\beta}::Indices(curved,position=fixed).
{\mu,\nu,\rho,\sigma,\kappa,\lambda,\alpha,\beta}::Integer(0..3).
```

D

```
e^{m \mu}::Vielbein.
e_{m \mu}::InverseVielbein.
g^{\mu\nu}::InverseMetric.
g_{\mu\nu}::Metric.
```

D

```
{ \epsilon,\psi_{\mu},\psi_{\mu\nu} }::Spinor(dimension=4, type=Majorana).
\Gamma_{#{m}}::GammaMatrix(metric=\delta).
{ \psi_{\mu\nu}, \psi_{\mu}, \epsilon }::AntiCommuting.
{ \psi_{\mu}, \psi_{\mu\nu} }::SelfAntiCommuting.
{ \epsilon, \psi_{\mu}, \psi_{\mu\nu} }::SortOrder.
\Gamma_{#}::Depends(\bar).
\psi_{\mu\nu}::AntiSymmetric.
```

I n p

```
L:= -1/2 e e^{n \nu} e^{m \mu} R_{\mu\nu n m}
              - 1/2 e \bar{\psi_\mu} \Gamma^{\mu\nu\rho} D_{\nu}{\psi_{\rho}}:
@rewrite_indices!(%){ \Gamma^{m n p} }{ e^{n \mu} };
```

$$L \quad \frac{1}{2}R_{\mu} \quad e^{\ m} \ e^{n} - \frac{1}{2}\bar{\psi}_{\mu}\Gamma^{m} \quad D_{\nu}\psi_{\rho}e^{\ m} \ e^{n} \ e^{p} \ ; \qquad : \ ( \qquad \nu$$

I n t

```
susy:= { e^{n \mu} -> -\bar{\epsilon} \Gamma^m \psi_\nu e^{m \mu} e^{n \nu},
         e          -> e \bar{\epsilon} \Gamma^n \psi_\mu e^{n\mu},
         \psi_\mu   -> D_{\mu}{\epsilon} }:
```

Var y
i

```
@vary!(L)( @(susy) );
```

$$
\begin{aligned}
L \quad & \frac{1}{2} R_{\mu} \quad \bar{\epsilon} \; {}^{p}\psi_{\rho} e \; {}^{p} \; e^{m} \; e^{n}_{\nu} \; + \frac{1}{2} R_{\mu} \quad e\bar{\epsilon} \; {}^{p}\psi_{\rho} e^{p} \; e^{m} \; e^{n}_{\nu} \qquad : \qquad \nu \\
& + \frac{1}{2} R_{\mu} \quad e \; {}^{m} \; \bar{\epsilon} \; {}^{p}\psi_{\rho} e^{p} \; e^{n}_{\nu} \; - \frac{1}{2} \Gamma^{m} \quad \overline{D_{\mu}\epsilon} \;\; {}_{\nu}\psi_{\rho} e \; {}^{m} \; e^{n} \; e^{p} \qquad \rho \\
& - \frac{1}{2} \Gamma^{m} \quad \overline{\psi_{\mu}} D_{\nu} D_{\rho}\epsilon \quad {}^{m} \; e^{n} \; e^{p} \; - \frac{1}{2} \Gamma^{m} \quad \overline{\psi_{\mu}} D_{\nu}\psi_{\rho}\bar{\epsilon} \; {}^{q}\psi_{\sigma} e \; {}^{q} \; e^{m} \; e^{n} \; e^{p} \qquad ( \qquad \sigma \; \nu \\
& + \frac{1}{2} \Gamma^{m} \quad \overline{\psi_{\mu}} D_{\nu}\psi_{\rho} e\bar{\epsilon} \; {}^{q}\psi_{\sigma} e^{q} \; e^{m} \; e^{n} \; e^{p} \; + \frac{1}{2} \Gamma^{m} \quad \overline{\psi_{\mu}} D_{\nu}\psi_{\rho} e \; {}^{m} \; \bar{\epsilon} \; {}^{q}\psi^{\mu}_{\sigma} e^{q} \; e^{n} \; e^{p} \qquad n \\
& + \frac{1}{2} \Gamma^{m} \quad \overline{\psi_{\mu}} D_{\nu}\psi_{\rho} e \; {}^{m} \; e^{n} \; \bar{\epsilon} \; {}^{q}\psi_{\sigma} e^{q} \; e^{p} \; ; \qquad \rho \qquad n \; \nu
\end{aligned}
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